What a fairer 24 team UEFA Euro could look like

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Format of the UEFA Euro

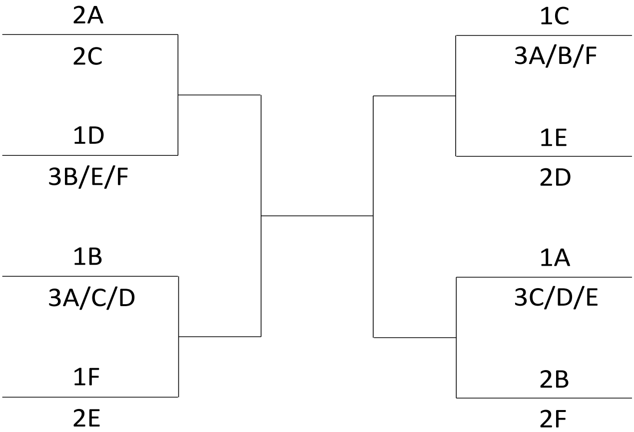
- Since 2016: **24 teams**
- Group stage (GS) + knockout stage (KO) starting with the round of 16
- Simplest reasonable symmetrical structure:
  - 4 groups of 6
  - Ro16: group winners play fourth-placed teams; group runners-up play third-placed teams
- Problem: 60 GS matches
  - Assuming 3 games per day, teams would play every 4th day, GS would last 3 weeks
  - Adding KO, tournament would last 5.5 weeks (more than the 32 team FIFA World Cup!)
  - Would not fit in the international calendar
- Other symmetrical structures:
  - 2 groups of 12: even worst...
  - 8 groups of 3: odd number of teams per group... (cf 1982, 2026+ FIFA World Cup)
- $\Longrightarrow$ UEFA has opted for **6 groups of 4**; 36 GS matches can be completed in 12 days

Problem: How to build a fair KO stage with 6 groups of 4?

- Number of groups is not a power of 2
- 16 teams must advance to the KO, ideally 16/6 teams per group should advance...
- UEFA has ruled that **the 6 group winners + the 6 runners-up + the 4 best third-placed teams** would advance
- In order to rank the 6 third-placed teams, UEFA considered in order: number of points obtained; goal difference; number of goals scored; fair play conduct in the final tournament; position in the UEFA national team coefficient rankings (see [2], article 18.03)

- **Asymmetry** $\Longrightarrow$ it is not obvious to devise a fair, balanced knockout bracket
- Reproducing what FIFA did for the 1986, 1990, and 1994 World Cups, UEFA chose for the Euro 2016 the following bracket

Bracket of the knockout stage of the UEFA Euro 2016



Third-placed teams allocation mechanism

	Official rule				All admissible alternative rules			
4 best 3rd	1A vs	1B vs	1C vs	1D vs	1A vs	1B vs	1C vs	1D vs
ABCD	3C	3D	3A	3B	3D	3C	3A	3B
ABCE	3C	3A	3B	3E	3E	3C	3A	3B
ABCF	3C	3A	3B	3F	3C	3A	3F	3B
ABDE	3D	3A	3B	3E	3E	3D	3A	3B
ABDF	3D	3A	3B	3F	3D	3A	3F	3B
ABEF	3E	3A	3B	3F	3E	3A	3F	3B
ACDE	3C	3D	3A	3E	3D	3C	3A	3E
ACDF	3C	3D	3A	3F	3D	3C	3A	3F
ACEF	3C	3A	3F	3E	3E	3C	3A	3F
ADEF	3D	3A	3F	3E	3E	3D	3A	3F
BCDE	3C	3D	3B	3E	3D	3C	3B	3E
BCDF	3C	3D	3B	3F	3C/D/D	3D/C/C	3F/B/F	3B/F/B
BCEF	3E	3C	3B	3F	3E	3C	3F	3B
BDEF	3E	3D	3B	3F	3E	3D	3F	3B
CDEF	3C	3D	3F	3E	3D	3C	3F	3E

Strengths of the bracket

- **Balance**:
  - Each half of the bracket has 3 group winners, 3 runners-up, and 2 third-placed teams
  - Each quarter of the bracket has 1 third-placed team, and either 2 group winners and 1 runner-up, or 1 group winner and 2 runners-up
  - Third-placed teams play against group winners in the round of 16
- **Group diversity**:
  - In each half of the bracket, the 3 group winners and the 3 runners-up come from the 6 different groups
  - In each quarter of the bracket, the 4 teams come from 4 different groups. This is what motivates the third-placed teams allocation mechanism
  - $\Longrightarrow$ winner and runner-up of any given group can only meet again in the final, and any two teams from any given group cannot meet again earlier than in the semifinals
  - Group diversity minimizes the probability of repeated matchups during the tournament

Flaws of the bracket

- **Group advantage**: In order to advance as far as possible in the tournament, it is an advantage/disadvantage to be drawn into some groups
  - It was a clear advantage to be drawn into Group A, and a clear disadvantage to be drawn into Group E
  - The fact that France was automatically placed into advantageous Group A has raised criticism, see [3, 1, 6]
- **Arbitrariness**:
  - Global structure of the bracket, i.e., distribution of the 3 following advantages:
    - Adv1: the winner plays against a third-placed team during Ro16 (4 groups)
    - Adv2: the runner-up plays against another runner-up during Ro16 (4 groups)
    - Adv3: the winner cannot play against another group winner before SF (2 groups) ($\Longrightarrow$ Adv1)
  - Third-placed teams allocation mechanism
- **Lack of win incentive**: For some groups, it is unclear whether it is better to finish first or second

The flaws of the UEFA Euro 2016 bracket

Group advantage

- **The worst case advantage** W measures, for a given group, the ease of the **most difficult** route to winning the tournament, averaged over the winner, runner-up, and third-placed team in the group:

$$W \equiv \frac{3}{8}\left(W_1 + W_2 + \frac{4}{6}W_3\right)$$

  - E.g., for Group A, $W_1 = 3 + 2 + 1 + 1 = 7$, $W_2 = 2 + 1 + 1 + 1 = 5$
  - For third-placed teams: $W_3 \equiv p_l W_3^l + p_r W_3^r$
- **The average advantage** A measures, for a given group, the ease of the **average** route to winning the tournament:

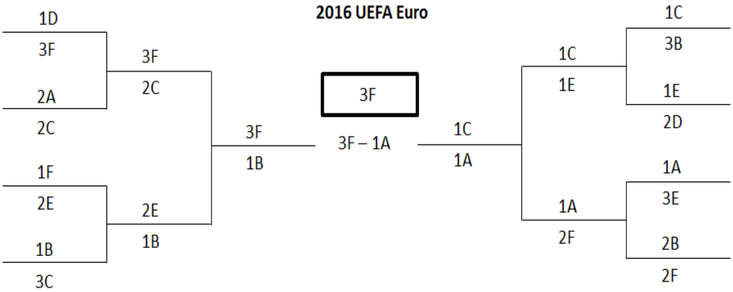
$$A \equiv \frac{3}{8}\left(A_1 + A_2 + \frac{4}{6}A_3\right)$$

  - E.g., for Group A,

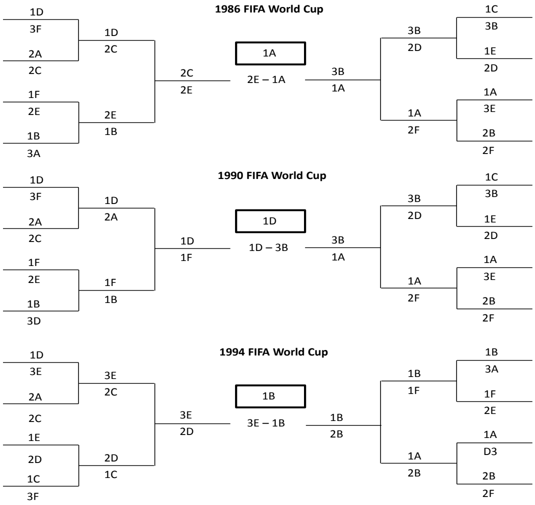
$$A_1 = 3 + \frac{1}{2}(2+2) + \frac{1}{4}(1+1+2+3) + \frac{1}{8}(1+1+1+2+2+2+3+3) = \frac{69}{8} = 8.625$$

Worst case advantage assumes that the best-ranked team always advances to the next round, while average advantage assumes that each team in the bracket has a 50% chance of advancing to the next round

Results of the knockout stage of the UEFA Euro 2016



Results of the knockout stages of the 1986, 1990, and 1994 FIFA World Cups



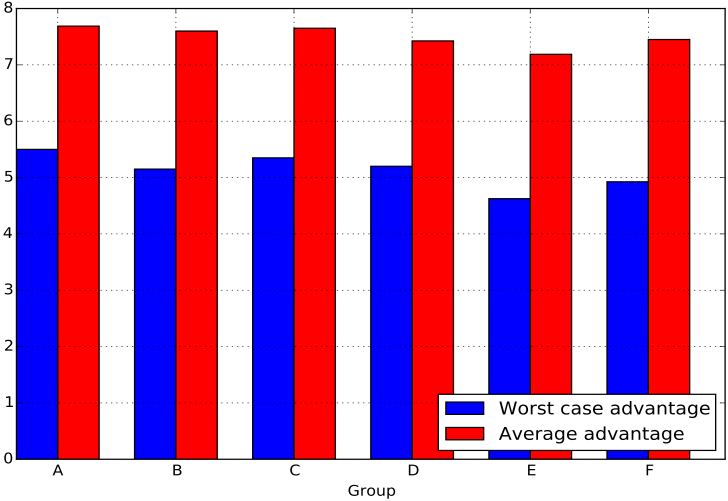
Statistics on the knockout stages of the 1986, 1990, and 1994 World Cups and the Euro 2016

Group ranks	1-2	1-3	2-3	Total
Number of games	19	22	5	46
Best ranked team advances	10	15	0	25
Ratio	52.6%	68.2%	0%	54.3%

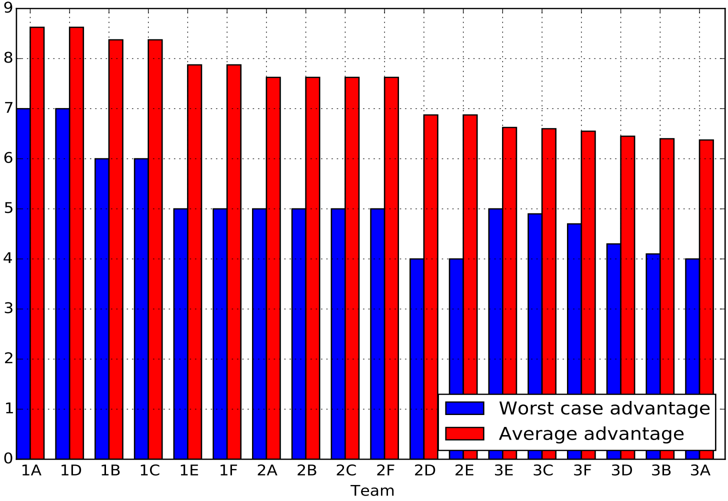
Group rank	1	2	3	Total
Nb teams reaching SF	8	4	4	16
Prob. of reaching SF	$\frac{8}{24} \simeq 33.3\%$	$\frac{4}{24} \simeq 16.7\%$	$\frac{4}{16} = 25\%$	$\frac{16}{64} = 25\%$

$\Longrightarrow$ **The average advantage is a more realistic measure of group advantage**

Group advantage



Worst case advantage and average advantage per team



Group	A	B	C	D	E	F
W_1	7	6	6	7	5	5
W_2	5	5	5	4	4	5
W_3 (See Table 2)	4	4.1	4.9	4.3	5	4.7
W	5.5	5.15	5.35	5.2	4.625	4.925
$W = W/4$	1.375	1.2875	1.3375	1.3	1.15625	1.23125
W_3' (See Table 2)	4	4.5	4.5	4.5	5	4.5
W'	5.5	5.25	5.25	5.25	4.625	4.875
$\bar{W}' = W'/4$	1.375	1.3125	1.3125	1.3125	1.15625	1.21875
A_1	8.625	8.375	8.375	8.625	7.875	7.875
A_2	7.625	7.625	7.625	6.875	6.875	7.625
A_3 (See Table 2)	6.375	6.4	6.6	6.45	6.625	6.55
A	7.6875	7.6	7.65	7.425	7.1875	7.45
$A = A/4$	1.921875	1.9	1.9125	1.85625	1.796875	1.8625
A_3' (See Table 2)	6.375	6.5	6.5	6.5	6.625	6.5
A'	7.6875	7.625	7.625	7.4375	7.1875	7.4375
$\bar{A}' = A'/4$	1.921875	1.90625	1.90625	1.859375	1.796875	1.859375

Table : Values of the worst case advantage and the average advantage for Groups A to F of the UEFA Euro 2016

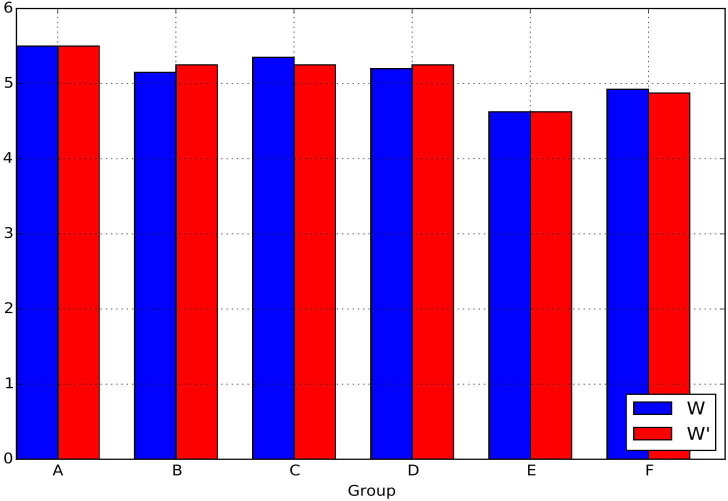
Group	A	B	C	D	E	F
Left side opponent of 3rd	1B	1D	1B	1B	1D	1D
p_l	0.7	0.1	0.1	0.7	0.7	0.7
W_3^l	4	5	4	4	5	5
A_3^l	6.375	6.625	6.375	6.375	6.625	6.625
Right side opponent of 3rd	1C	1C	1A	1A	1A	1C
p_r	0.3	0.9	0.9	0.3	0.3	0.3
W_3^r	4	4	5	5	5	4
A_3^r	6.375	6.375	6.625	6.625	6.625	6.375
$W_3 = p_l W_3^l + p_r W_3^r$	4	4.1	4.9	4.3	5	4.7
$A_3 = p_l A_3^l + p_r A_3^r$	6.375	6.4	6.6	6.45	6.625	6.55
$p'_l = p'_r$	0.5	0.5	0.5	0.5	0.5	0.5
$W'_3 = p'_l W_3^l + p'_r W_3^r$	4	4.5	4.5	4.5	5	4.5
$A'_3 = p'_l A_3^l + p'_r A_3^r$	6.375	6.5	6.5	6.5	6.625	6.5

Table : Values of p_l, W_3^l, A_3^l, p_r, W_3^r, A_3^r, W_3, A_3, p'_l, p'_r, W'_3, and A'_3 for Groups A to F.

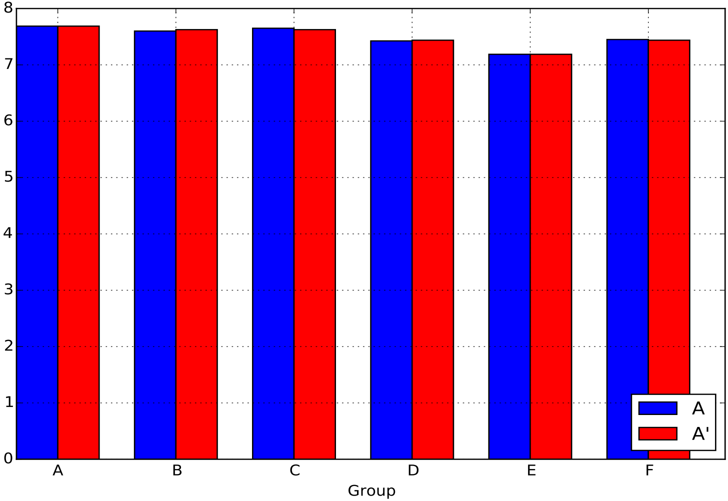
Arbitrariness

- Two types of arbitrary choices:
  - Global structure of the bracket: repartition of the 4 Adv1, the 4 Adv2, and the 2 Adv3
  - Placement of the third-placed teams in the bracket
- Most of the group advantage effect is explained by the arbitrary global structure of the bracket
- However, part of it also results from the particular, arbitrary allocation of third-placed teams
- To disentangle between the two, we consider what the group advantage would have been if third-placed teams had been placed in the bracket in a symmetric way, i.e., $p_l = p_r$ (denoted with primes)
  - either by picking (arbitrarily) one of the exactly 1,000 symmetric allocation rules (out of $2^{16} = 65,536$)
  - or by drawing uniformly one of the 2 (or 4) admissible allocation rules, once the 4 best third-placed teams are known

Comparison of W and W'



Comparison of A and A'



Lack of win incentive

- For Group F, $W_1 = W_2$, a sign of bad tournament design; $A_1 - A_2 > 0$ but small
- For Groups D and E, $W_3 > W_2$; $A_2 - A_3 > 0$ but small. Note, however, that it is risky to finish third in the group if the qualification of the third-placed team is not secured yet.

Can we build a better bracket based only on group ranks?

Can we build a better bracket based only on group ranks?

- Predetermined balanced bracket routes based only on group ranks
- Better = group advantage would be minimized
- Remember:
  - Adv1: winner plays against a third-placed team during Ro16 (4 groups)
  - Adv2: runner-up plays against another runner-up during Ro16 (4 groups)
  - Adv3: winner cannot play against another group winner before SF (2 groups) ($\Longrightarrow$ Adv1)
- Can we make sure that no group benefits from Adv1-Adv2-Adv3 without sacrificing group diversity? **No**
- Adv2 without Adv1 (like Group F in 2016) = both the winner and the runner-up play against a runner-up during Ro16: pb of win incentive. Only way to avoid this: enforce that Adv2 $\Longrightarrow$ Adv1. Very unfair: 2 groups would benefit from none of the 3 advantages, and 2 from all 3

$\Longrightarrow$ **Group advantage cannot be avoided in a format with predetermined balanced bracket routes that are based only on group ranks and satisfy group diversity**

New fairer brackets

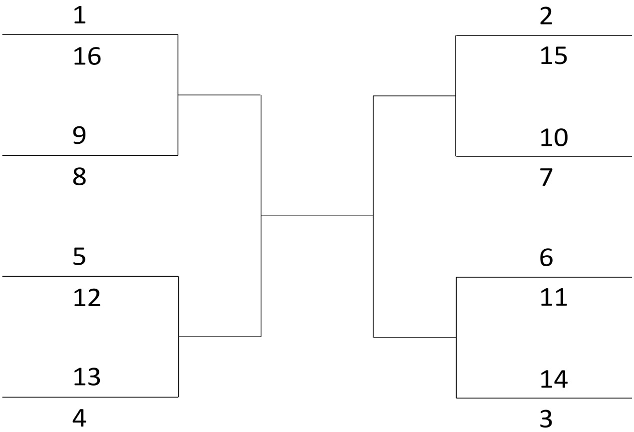
New fairer brackets

- We keep intact the format of the group stage (6 groups of 4), as well as the strengths of the current bracket: balance and group diversity
- Goals: **Eliminate group advantage, increase win incentive, minimize arbitrary choices**
- We suggest two new fairer brackets that use **global rankings 1–16** instead of only group ranks:
  - 1 = best group winner, 2 = second best group winner, . . ., 6 = lowest ranked group winner
  - 7 = best runner-up, 8 = second best runner-up, . . ., 12 = lowest ranked runner-up
  - 13, 14, 15, and 16 = four best third-placed teams

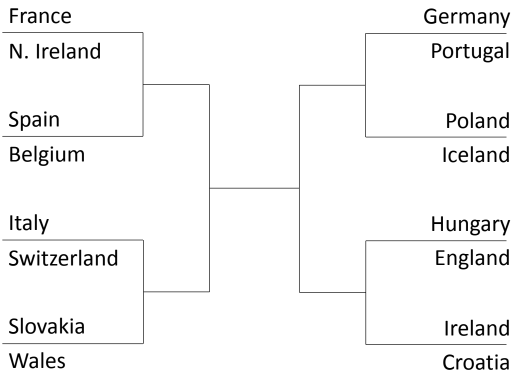
Global rankings 1–16 after the group stage of the UEFA Euro 2016

Rk	Team	Group	Pts	GD	GF
Group winners					
1	France	$G_1 = \mathrm{A}$	7	+3	4
2	Germany	$G_2 = \mathrm{C}$	7	+3	3
3	Croatia	$G_3 = \mathrm{D}$	7	+2	5
4	Wales	$G_4 = \mathrm{B}$	6	+3	6
5	Italy	$G_5 = \mathrm{E}$	6	+2	3
6	Hungary	$G_6 = \mathrm{F}$	5	+2	6
Runners-up					
7	Poland	$G_7 = \mathrm{C}$	7	+2	2
8	Spain	$G_8 = \mathrm{D}$	6	+3	5
9	Belgium	$G_9 = \mathrm{E}$	6	+2	4
10	Iceland	$G_{10} = \mathrm{F}$	5	+1	4
11	England	$G_{11} = \mathrm{B}$	5	+1	3
12	Switzerland	$G_{12} = \mathrm{A}$	5	+1	2
Third-placed teams					
13	Slovakia	$G_{13} = \mathrm{B}$	4	0	3
14	Ireland	$G_{14} = \mathrm{E}$	4	-2	2
15	Portugal	$G_{15} = \mathrm{F}$	3	0	4
16	N. Ireland	$G_{16} = \mathrm{C}$	3	0	2

Ideal bracket



Ideal bracket if we use the Euro 2016 rankings



Slightly distorting the ideal bracket

Problem: this bracket does not satisfy the group diversity constraint

We suggest two ways of minimally distorting the ideal bracket:

- **A small deterministic distortion of the ideal bracket**:
  - The position of each group winner (teams 1 to 6) in the bracket is kept intact
  - The positions of the runners-up, as well as the positions of the third-placed teams, are shuffled in a deterministic way to ensure group diversity and foster win incentive
- **A small random distortion of the ideal bracket**:
  - Alternatively, we can slightly randomize the ideal bracket in a way that ensures group diversity and preserves balance, win incentive, and absence of group advantage
  - $\Longrightarrow$ A new draw would be organized, right at the end of the group stage, in order to decide the final bracket

A new fairer deterministic bracket

We denote by G_i the group (A to F) of team i, $1 \leq i \leq 16$

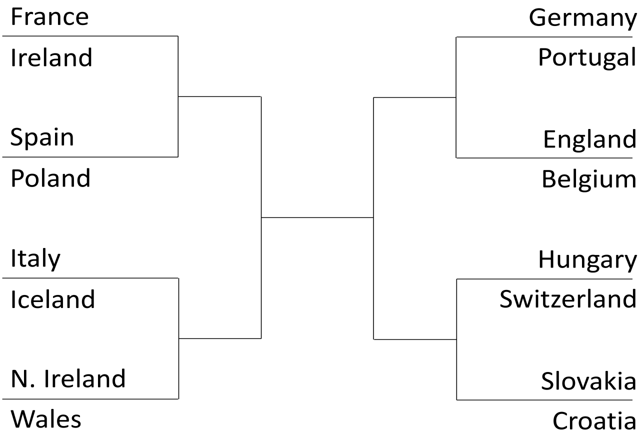
- Position of each group winner (teams 1 to 6) in the bracket is kept intact
- First look at the groups of teams 1, 4, and 5 (lhs). Group diversity $\Longrightarrow$ runners-up of these 3 groups must be placed on rhs. Win incentive $\Longrightarrow$ lowest ranked of these 3 runners-up plays against team 6, a group winner. The other 2 runners-up play against each other (positions 7 and 10 of ideal bracket). Symmetrically for the groups of teams 2, 3, and 6 (rhs)
- The 4 third-placed teams must be placed in the 4 remaining spots (positions 13, 14, 15, 16 of the ideal bracket) in a way that guarantees group diversity. $135 = 3 \times 3 \times 15$ configurations to consider: 3 possible cases for group of lowest ranked right runner-up (G_1, G_4, or G_5), 3 possible cases for group of lowest ranked left runner-up (G_2, G_3, or G_6), and 15 possible combinations of the 4 third-placed teams. Among those 135 configurations, there are only 6 unfavorable configurations where it is impossible to satisfy group diversity: when lowest ranked of 3 right runners-up is from G_1, lowest ranked of 3 left runners-up is from G_2, and 3 of the 4 best third-placed teams come from groups G_1, G_4, G_5 or from groups G_2, G_3, G_6

- Favorable cases (129 out of 135): there exists 2 (in 112 cases out of 129) or 4 (in 17 cases out of 129) admissible allocations of the 4 best third-placed teams
- Unfavorable cases (6 cases out of 135): instead of playing against the lowest ranked of the 3 right runners-up, team 6 would play against the middle-ranked right runner-up. Then we would be back in the situation where there exists 2 (in 28 cases out of 30) or 4 (in 2 cases out of 30) admissible allocations of the 4 best third-placed teams

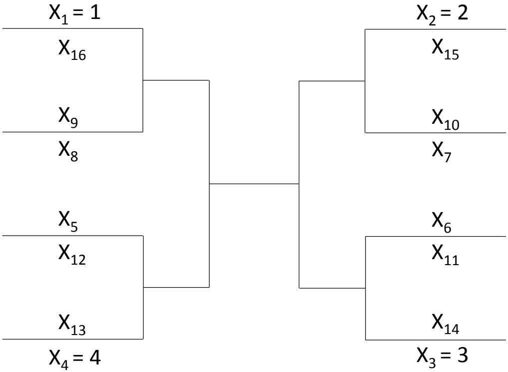
Finally we must pick 1 of the 2 or 4 admissible allocations. Among the 2 or 4 admissible allocations, we keep the ones where the opponent of team 1 has the lowest rank:

- If only one allocation is left, we use it to define the final bracket
- If not, then there are exactly 2 admissible allocations left, both have different opponents for team 2, and to define the final bracket we use the one with the lowest ranked opponent of team 2

Bracket produced by our suggested deterministic procedure if we use the Euro 2016 rankings



A new fairer random bracket, case 1: $\{G_7, G_8\} \neq \{G_1, G_4\}$ and $\{G_7, G_8\} \neq \{G_2, G_3\}$



$X_i =$ team in position i of the ideal bracket in our random bracket, $1 \leq i \leq 16$

A new fairer random bracket, case 1: $\{G_7, G_8\} \neq \{G_1, G_4\}$ and $\{G_7, G_8\} \neq \{G_2, G_3\}$

Set $X_1 = 1$, $X_2 = 2$, $X_3 = 3$, and $X_4 = 4$. We will draw a combination of teams $X_5, \ldots, X_{16}$ that satisfies the following constraints:

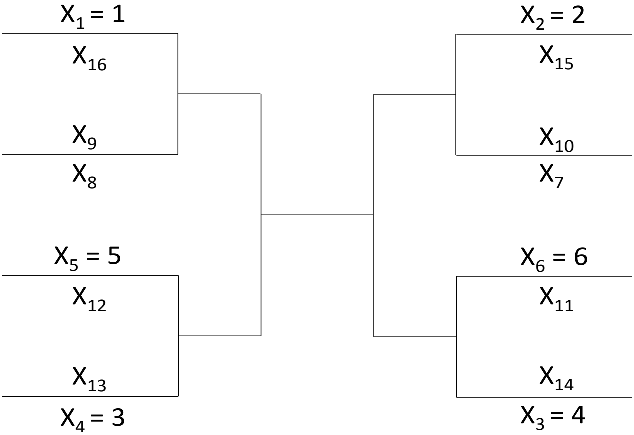
- $\{X_5, X_6\} = \{5, 6\}$
- $\{X_7, X_8\} = \{7, 8\}$
- $\{X_9, X_{10}, X_{11}, X_{12}\} = \{9, 10, 11, 12\}$
- $\{X_{13}, X_{14}, X_{15}, X_{16}\} = \{13, 14, 15, 16\}$
- $\#\{G_{X_1}, G_{X_4}, G_{X_5}, G_{X_8}, G_{X_9}, G_{X_{12}}\} = 6$
- $\#\{G_{X_2}, G_{X_3}, G_{X_6}, G_{X_7}, G_{X_{10}}, G_{X_{11}}\} = 6$
- $G_{X_{16}} \notin \{G_{X_1}, G_{X_8}, G_{X_9}\}$
- $G_{X_{15}} \notin \{G_{X_2}, G_{X_7}, G_{X_{10}}\}$
- $G_{X_{14}} \notin \{G_{X_3}, G_{X_6}, G_{X_{11}}\}$
- $G_{X_{13}} \notin \{G_{X_4}, G_{X_5}, G_{X_{12}}\}$

The first 4 constraints ensure that the bracket is balanced; the last 6 constraints ensure that group diversity is satisfied

A new fairer random bracket, case 1: $\{G_7, G_8\} \neq \{G_1, G_4\}$ and $\{G_7, G_8\} \neq \{G_2, G_3\}$

- If $\{G_7, G_8\} \neq \{G_1, G_4\}$ and $\{G_7, G_8\} \neq \{G_2, G_3\}$, the number N of combinations of teams $X_5, \ldots, X_{16}$ that satisfy all the constraints above belongs to $\{6, 8, 10, 12, 16, 18, 20, 24\}$
- When the group stage is over, teams 1 to 16 would be known, and if $\{G_7, G_8\} \neq \{G_1, G_4\}$ and $\{G_7, G_8\} \neq \{G_2, G_3\}$, then the exhaustive list of the N admissible brackets (i.e., admissible combinations of teams $X_5, \ldots, X_{16}$) would be published, and one of the N brackets would be randomly drawn, uniformly

A new fairer random bracket, case 2: $\{G_7, G_8\} = \{G_1, G_4\}$ or $\{G_7, G_8\} = \{G_2, G_3\}$



A new fairer random bracket, case 2: $\{G_7, G_8\} = \{G_1, G_4\}$ or $\{G_7, G_8\} = \{G_2, G_3\}$

- The only situation where $N = 0$ is when $\{G_7, G_8\} = \{G_1, G_4\}$ or $\{G_7, G_8\} = \{G_2, G_3\}$. Indeed, if $\{G_7, G_8\} = \{G_1, G_4\}$, for any permutation (X_7, X_8) of teams $(7, 8)$, the runner-up X_8 must come from the same group as that of a group winner (team 1 or team 4), in the left half of the bracket. Symmetrically, if $\{G_7, G_8\} = \{G_2, G_3\}$, for any permutation (X_7, X_8) of teams $(7, 8)$, the runner-up X_7 must come from the same group as that of a group winner (team 2 or team 3), in the right half of the bracket.
- In this situation, we suggest to set $X_1 = 1$, $X_2 = 2$, $X_3 = 4$, $X_4 = 3$, $X_5 = 5$ and $X_6 = 6$. Then the number N of combinations of teams $X_7, \ldots, X_{16}$ that satisfy all the constraints above belongs to $\{8, 10\}$. Like in case 1, we would then simply draw one of the N admissible brackets randomly, uniformly, to decide the final bracket

France (X_1) - Slovakia (X_{16})	Germany (X_2) - Portugal (X_{15})
Poland (X_8) - Iceland (X_9)	Spain (X_7) - Belgium (X_{10})
Italy (X_5) - England (X_{12})	Hungary (X_6) - Switzerland (X_{11})
Croatia (X_4) - N. Ireland (X_{13})	Wales (X_3) - Ireland (X_{14})

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Italy (X_5) - England (X_{12})	Hungary (X_6) - Belgium (X_{11})
Croatia (X_4) - Portugal (X_{13})	Wales (X_3) - N. Ireland (X_{14})

France (X_1) - Ireland (X_{16})	Germany (X_2) - Slovakia (X_{15})
Poland (X_8) - Iceland (X_9)	Spain (X_7) - Switzerland (X_{10})
Italy (X_5) - England (X_{12})	Hungary (X_6) - Belgium (X_{11})
Croatia (X_4) - Portugal (X_{13})	Wales (X_3) - N. Ireland (X_{14})

France (X_1) - Ireland (X_{16})	Germany (X_2) - Portugal (X_{15})
Poland (X_8) - England (X_9)	Spain (X_7) - Switzerland (X_{10})
Italy (X_5) - Iceland (X_{12})	Hungary (X_6) - Belgium (X_{11})
Croatia (X_4) - Slovakia (X_{13})	Wales (X_3) - N. Ireland (X_{14})

France (X_1) - Portugal (X_{16})	Germany (X_2) - Ireland (X_{15})
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Croatia (X_4) - Slovakia (X_{13})	Wales (X_3) - N. Ireland (X_{14})

Benefits and drawbacks of the new suggested systems

- **Balance** and **group diversity** are enforced
- **Group advantage has been eliminated**
- Global win incentive: due to the global structure of the bracket, it is better to win the group than to be the runner-up, and to be the runner-up than to be in third place
- Win incentive is deeper than that: with our suggested deterministic bracket, all teams have an incentive to score a lot of goals, even if that does not change their ranking in the group, as it can improve their ranking within group winners, runners-up, or third-placed teams. Cf Italy, France, Spain during the Euro 2016 $\Longrightarrow$ **Using global rankings 1–16 to build the knockout bracket would significantly increase win incentive, as well as interest and excitement for the group stage**
- The suggested random system involves a new draw ceremony, which would take place right after the last matches of the group stage are finished. This could be appealing, as the draw would lend itself to a nice, widely anticipated TV show of about 30 minutes

Benefits and drawbacks of the new suggested systems

- Given that group labels A to F would be totally unrelated to the final bracket, seeded teams could now be allocated to Groups A to F without impacting the knockout stage. For instance, **seeded teams could be allocated to groups based on geographical criteria**. Even though we would not recommend it in view of sporting fairness, UEFA may find this possibility interesting in view of maximizing ticket sales and maximizing the presence of fans of seeded teams.[1]
- **Logistic issues**: all teams would need to wait until the end of the group stage to know their opponent/stadium in Ro16 and their possible opponents in future rounds. However, this would improve sporting fairness, by placing all teams on an equal foot

[1] For the 2015 Women's World Cup, which also featured 24 teams, FIFA allocated the 6 seeded teams to groups A to F *before* the draw. This meant that FIFA almost *decided* that France and Germany would meet in quarterfinals, as they placed Germany in Group B and France in Group F, which meant that if both teams won their group and advanced to the quarterfinals, they would play against each other – which is exactly what happened. This was, of course, a terrible way of organizing the tournament, and proved how difficult it is for FIFA to cope with sporting fairness.

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