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Analysis of a Constructive Matheuristics for the Traveling Umpire Problem

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Round 1	A x E	F x B	G x C	D x H
Round 2	A x F	B x H	E x C	D x G
		• • •		
Round 4n -2	E x A	D x B	H x C	F x G

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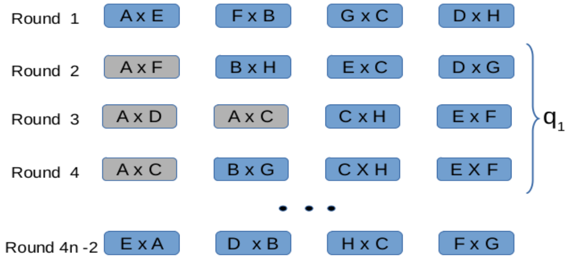

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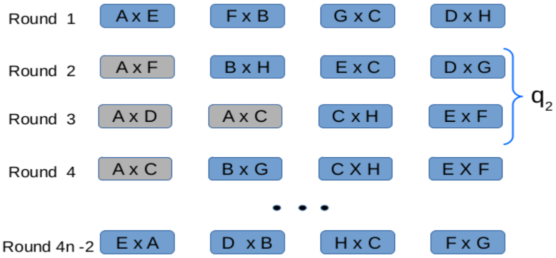
4. An umpire cannot visit the same venue more than once in any $q1$ consecutive rounds
5. An umpire cannot officiate the games of the same team more than once in any $q2$ consecutive rounds.



Constructive Matheuristics

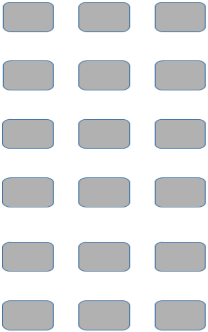
Matheuristics

Matheuristics are heuristic algorithms which are hybrids of metaheuristic and mathematical programming algorithms.

Constructive Matheuristics: The idea

- Decompose the problem into subproblems
- Solve the subproblems using exact methods
- Combine the solutions to construct a solution to the entire problem

Constructive Matheuristic procedure



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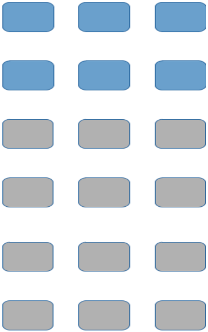

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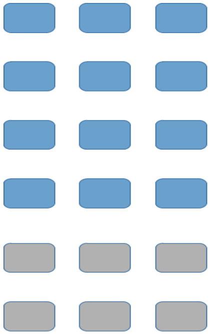
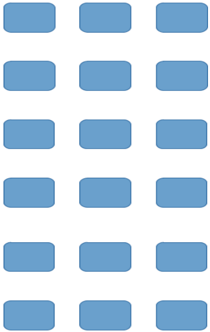
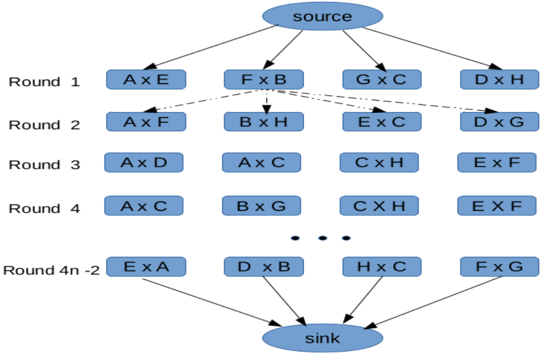


Constructive Matheuristic procedure



Flow formulation of the problem



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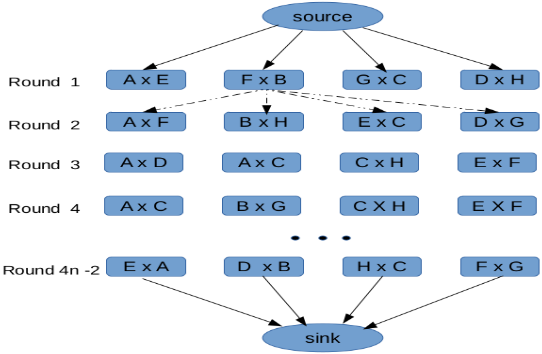
Flow formulation of the problem



d_e : distance of directed edge e

$$X_{eu} = \begin{cases} 1 & \text{if edge } e \text{ is selected for umpire } u \\ 0 & \text{otherwise} \end{cases}$$

Flow formulation of the problem

$$\text{minimize:} \quad \sum_{e \in E} \sum_{u \in U} d_e X_{eu} \tag{1}$$

$$\text{subject to:} \quad \sum_{e \in \delta(j)} \sum_{u \in U} X_{eu} = 1 \quad \forall j \in V \backslash \{\text{source, sink}\} \tag{2}$$

$$\sum_{e \in \delta(j)} X_{eu} - \sum_{e \in \omega(j)} X_{eu} = \begin{cases} -1 & \text{if } j \text{ is the source} \\ +1 & \text{if } j \text{ is the sink} \\ 0 & \forall j \in V \backslash \{\text{source, sink}\} \end{cases}, \quad \forall u \in U \tag{3}$$

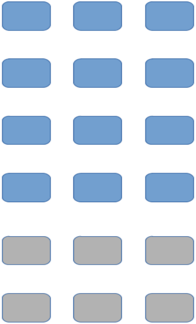
$$\sum_{e \in \delta(H_i)} X_{eu} \geq 1 \quad \forall i \in I, \forall u \in U \tag{4}$$

$$\sum_{e \in \delta(Q'_{ir})} X_{eu} \leq 1 \quad \forall i \in I, \forall r \in R, \forall u \in U \tag{5}$$

$$\sum_{e \in \delta(Q''_{ir})} X_{eu} \leq 1 \quad \forall i \in I, \forall r \in R, \forall u \in U \tag{6}$$

$$X_{eu} \in \{0, 1\} \quad \forall e \in E, \forall u \in U \tag{7}$$

Constructive Matheuristic procedure



Pitfall: Constraint 3 can be evaluated only on a full schedule

Constructive Matheuristic procedure

Pitfall: Constraint 3 can be evaluated only on a full schedule

$$Obj = \sum_{u \in U} \sum_{e \in E_b} (X_{ue} d_e - \sqrt{b_0} w X_{ue} y_{ue})$$

where,

$$E_b : \text{Edges in block } b \tag{8}$$

$$b_0 : \text{start round of block } b \tag{9}$$

$$y_{ue} = \begin{cases} 1 & \text{edge } e \text{ starts at a node which has not been visited by umpire } u \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

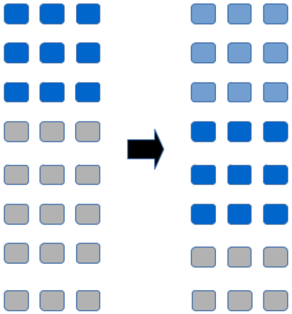
Design parameters

- Weight (w)

$$Obj = \sum_{u \in U} \sum_{e \in E_b} (X_{ue} d_e - \sqrt{b_0} \mathbf{w} X_{ue} y_{ue})$$

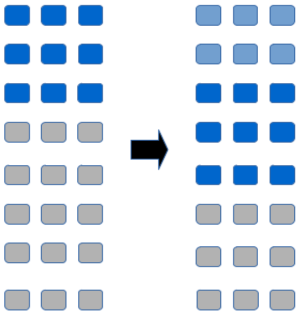
Design parameters

- Weight (w)
- Block size (β)



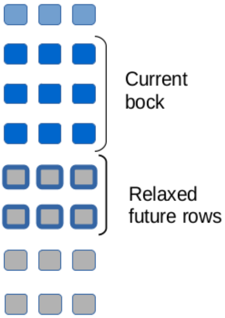
Design parameters

- Weight (w)
- Block size (β)
- Overlap (θ)

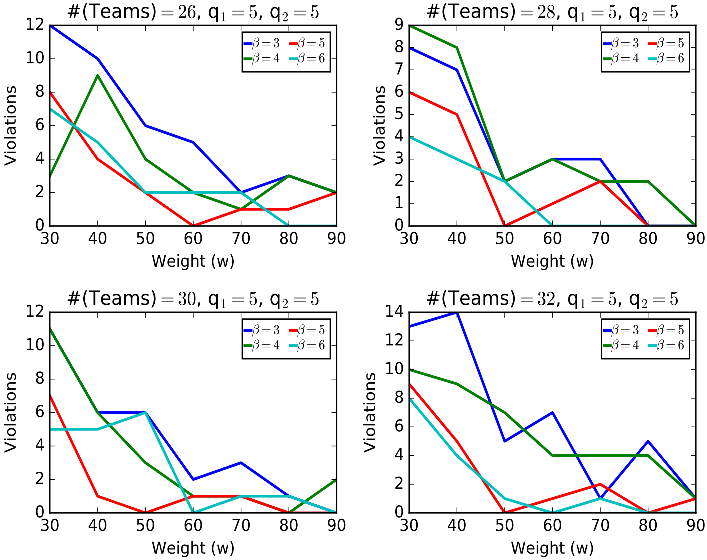


Design parameters

- Weight (w)
- Block size (β)
- Overlap (θ)
- relaxed future rows

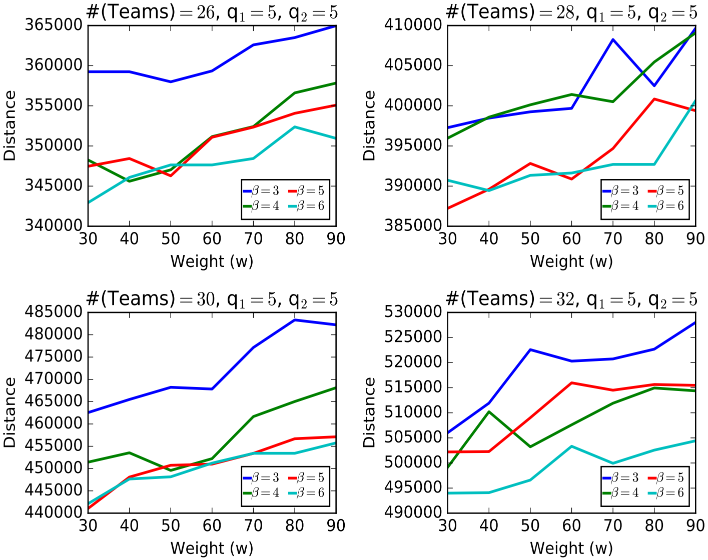


Weight(w)

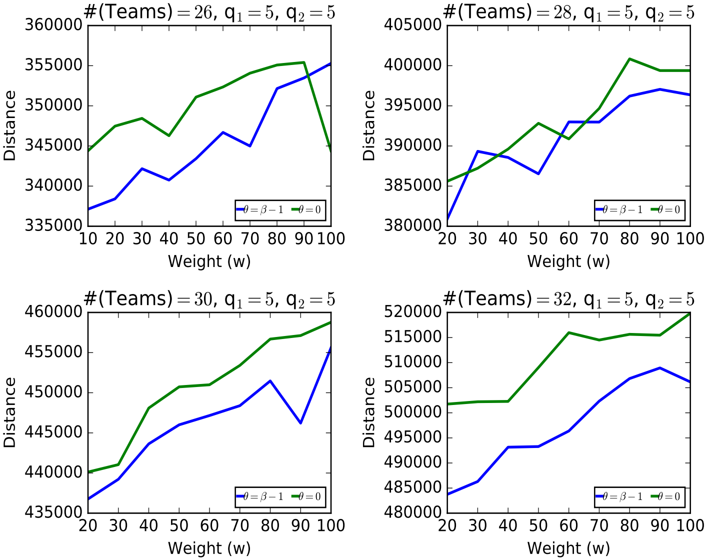


#(Teams) = 26, $q_1 = 5$, $q_2 = 5$
#(Teams) = 28, $q_1 = 5$, $q_2 = 5$
#(Teams) = 30, $q_1 = 5$, $q_2 = 5$
#(Teams) = 32, $q_1 = 5$, $q_2 = 5$
Violations
Weight (w)
$\beta = 3$
$\beta = 4$
$\beta = 5$
$\beta = 6$

Block size (β)



Overlap (θ) and Distance



Overlap (θ) and Violations (k)

w	$\beta = 3$			$\beta = 4$			$\beta = 5$			$\beta = 6$		
	Obj	k	time(s)	Dist.	k	time(s)	Dist.	k	time(s)	Dist.	k	time(s)
10	444087	22	22.67	435978	15	163.26	430773	19	7379.96	435535	19	32453.73
20	445050	12	22.45	438341	8	185.35	436760	7	4360.58	436869	10	20367.06
30	452115	8	23.31	444602	1	169.58	439221	2	6267.70	435723	2	21211.50
40	453230	1	26.69	450080	2	193.89	443632	0	5912.56	441140	1	19736.96
50	453811	0	19.66	447684	1	212.82	446004	0	3252.25	444635	0	16354.74
60	455028	1	21.19	453836	0	222.77	447160	0	3528.40	446980	0	22174.10
70	457514	1	22.08	451806	0	189.73	448388	0	3387.97	447505	0	19274.77
80	463096	0	19.83	454258	0	193.68	451471	0	2802.27	447059	0	13875.05
90	468985	0	20.14	460429	0	188.09	446214	0	3269.73	456360	0	21546.72
100	468561	0	20.94	465034	0	243.00	455599	0	2992.34	455202	0	20270.37

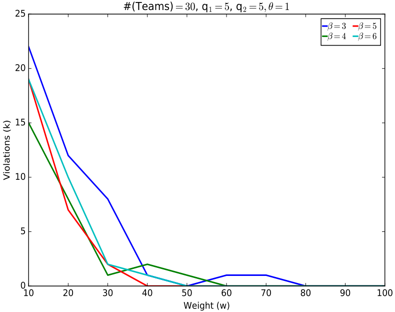


Table: Results for large instances

#(Teams)	q1	q2	Dist.	k	β	w	$\beta - \theta$	Time (s)	Best Solution	% Gap
26	13	6								
26	5	5	345340	0	6	50	2	2863.172		
28	14	7								
28	5	5	390933	0	6	50	2	9266.189		
30	15	7								
30	5	5	441931	0	6	50	2	7946.105	450919	-1.993
32	16	8								
32	5	5	499305	0	5	60	2	7819.437		

Benchmark website: http://gent.cs.kuleuven.be/tup

Table: Results for medium instances

#(Teams)	q1	q2	Dist.	β	w	$\beta - \theta$	Time (s)	Best Solution	% Gap
12	7	2	86889	8	10	1	4.399	86889	0
12	5	3	93679	6	20	1	2.276	93679	0
12	4	3	90254	8	80	1	11.196	89826	0.476
14	8	3	183631	9	90	1	310.402	172177	6.652
14	8	2	149017	7	50	1	7.311	147824	0.807
14	7	3	164440	10	10	2	94.359	164440	0
14	7	2	146980	8	20	1	18.55	146656	0.221
14	6	3	159507	10	100	1	312.184	158875	0.398
14	6	2	145414	10	60	5	17.878	145124	0.2
14	5	3	156780	8	90	4	13.925	154962	1.173
14	5	2	143548	8	20	3	6.983	143357	0.133
14A	8	3	170026	9	40	6	34.282	166184	2.312
14A	8	2	144165	9	80	1	27.473	143043	0.784
14A	7	3	160830	10	100	5	137.762	158760	1.304
14A	7	2	141673	8	10	1	19.548	140562	0.79
14A	6	3	154402	8	90	4	13.045	152981	0.929
14A	6	2	139775	6	30	3	2.824	138927	0.61
14A	5	3	149751	8	60	1	59.256	149331	0.281
14A	5	2	137853	7	60	2	5.699	137853	0
14B	8	3	168107	6	10	1	6.145	165026	1.867
14B	8	2	141628	10	10	1	72.471	141312	0.224
14B	7	3	157884	9	10	1	152.308	157884	0
14B	7	2	139208	5	20	1	2.284	138998	0.151
14B	6	3	154275	7	60	1	28.649	152740	1.005
14B	6	2	138297	8	30	2	18.611	138241	0.041
14B	5	3	151015	8	90	3	30.138	149455	1.044
14B	5	2	136245	8	70	3	18.003	136069	0.129

Table: Results for medium instances

#(Teams)	q1	q2	Dist.	β	w	$\beta - \theta$	Time (s)	Best Solution	% Gap
14C	8	3	178310	7	70	1	26.04	161262	10.572
14C	8	2	141998	8	10	1	14.532	141015	0.697
14C	7	3	159812	8	20	1	85.207	154913	3.162
14C	7	2	138928	5	70	1	1.638	138832	0.069
14C	6	3	153541	6	80	1	9.407	150858	1.778
14C	6	2	136913	5	30	1	2.133	136394	0.381
14C	5	3	149459	7	60	3	9.2	148349	0.748
14C	5	2	135302	5	30	1	2.387	134916	0.286
16	8	3	no sol					189415	
16	8	2	168604	8	90	3	117.139	161999	4.077
16	7	4	no sol					197028	
16	7	3	168585	7	40	2	56.442	165765	1.701
16	7	2	152563	7	40	1	120.076	150433	1.416
16A	8	3	no sol					214512	
16A	8	2	179692	7	60	2	19.325	171882	4.544
16A	7	4	no sol					213416	
16A	7	3	180641	8	50	2	311.647	178511	1.193
16A	7	2	166224	7	10	1	71.294	163709	1.536
16B	8	3	no sol					217764	
16B	8	2	no sol					180728	
16B	7	4	no sol					223868	
16B	7	3	184636	8	50	1	6003.46	180204	2.459
16B	7	2	169565	8	50	3	143.669	167190	1.421

Table: Results for medium instances

#(Teams)	q1	q2	Dist.	β	w	$\beta - \theta$	Time (s)	Best Solution	% Gap
16C	8	3	no sol.					214993	
16C	8	2	186437	6	80	3	4.318	179939	3.611
16C	7	4	no sol					209088	
16C	7	3	185769	8	90	3	1067.693	180483	2.929
16C	7	2	169466	7	50	1	145.683	166479	1.794
18	9	4	no sol.					267311	
18	9	3	no sol.					262987	
18	8	4	no sol.					254155	
18	8	3	204658	6	50	3	21.191	248302	-17.577
18	7	4	216051	6	60	2	74.843	217502	-0.667
20	10	5	no sol.						
22	11	5	no sol.						

Table: Some best results

#(Teams)	q1	q2	Dist.	β	w	$\beta - \theta$	Best Solution	% Gap
18	8	3	202662	8	10	2	248302	-18.380
18	7	4	215129	7	20	4	217502	-1.091
26	5	5	345340	6	50	2		
28	5	5	386528	5	50	1		
30	5	5	441931	6	50	2	450919	-1.993
32	5	5	495338	5	50	3		

Future work

- Study interaction of design parameters
- Experiments on relaxed future
- Order of execution of subproblems
- Comparison of various formulations for the TUP

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THANK YOU!